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Heat transfer from a hypersonic turbulent boundary layer on a flat plate

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A hypersonic gun tunnel has been used to measure the heat transfer to a sharp-edged flat plate inclined at various incidences to generate local Mach numbers from 3 to 9. The measurements have been compared with a number of theoretical estimates by plotting the Stanton number against the energy-thickness Reynolds number. The prediction giving the most reasonable agreement throughout the above Mach number range is that due to Fernholz (1971).

The values of the skin-friction coefficient derived from velocity profiles and Preston tube data are also given.

1. Introduction

In the field of hypersonic turbulent boundary layers there are now an embarrassingly large number of prediction techniques chasing too few experimental data. A recent note by Harvey & Clark (1972) compared some measured null-type skin-friction balance measurements at $M = 20$ with eight flat-plate prediction methods commonly used for compressible turbulent flow. Discrepancies between measured and predicted values ranged from -40 to $+125\,\%$. Since many of the prediction techniques are themselves based on data correlations there is no room for complacency in either the theoretical or experimental camp. Clearly there are wide variations in the various predictions and considerable scatter in the experimental data.

Our modest aim has been to make direct measurements of heat transfer to an isothermal cold wall at local Mach numbers between 3 and 9. At the higher Mach numbers a region of laminar flow was followed by a long transition zone before turbulent flow was established. Therefore some tests were made using vortex generators near the leading edge to trip the flow. Some Pitot pressure profile and Preston tube measurements were also made in order to estimate the skin-friction coefficient and hence derive a value for the Reynolds analogy factor F, where F is the ratio of the Stanton number St to half the skin-friction coefficient C_f.

2. Data presentation

It is well known that, if the Stanton number or skin-friction coefficient is plotted against the length Reynolds number, experiment and theory can usually be made to agree by the judicious choice of a virtual origin, so this approach was discarded. To plot the data against the momentum-thickness Reynolds number would have meant either calculating the momentum thickness θ from Mach number and total temperature profiles or using a Reynolds analogy factor to derive θ from the energy thickness Γ. Both these methods are fraught with difficulty owing to the current uncertainty surrounding both total-temperature probe measurements and the numerical value of F under hypersonic cold-wall conditions. Instead the technique of plotting St *vs.* Re_Γ has been adopted as suggested by Hopkins *et al.* (1969). Both quantities are defined using conditions at the edge of the boundary layer (denoted by a subscript e) thus

$$St \equiv \dot{q}/\rho_e u_e (H_r - H_w),$$

where $\dot{q}$ is the heat-transfer rate at the wall, H_r the recovery enthalpy and H_w its value at the wall. A recovery factor of 0·9 has been used throughout. Re_Γ is calculated from $Re_\Gamma = \rho_e u_e \Gamma / \mu_e$.

If the energy thickness (a better term is the enthalpy defect thickness) Γ is defined by

$$\Gamma = \int_0^\infty \frac{\rho u}{\rho_e u_e} \left| \frac{H_e - H(y)}{H_e - H_w} \right| dy, \tag{1}$$

where H is the total enthalpy and the other symbols are standard, then the integral form of the energy equation (heat balance) may be written as

$$d\{\rho_e u_e |H_e - H_w| \,\Gamma\}/dx = \dot{q}. \tag{2}$$

Thus Γ can be found directly from measurement of the heat-transfer-rate distribution $\dot{q}(x)$ as

$$\Gamma = \{\rho_e u_e |H_e - H_w|\}^{-1} \int_0^x \dot{q}\, dx \tag{3}$$

and an energy-thickness Reynolds number for two-dimensional flow may be defined unambiguously and without reference to any virtual origin.

3. Apparatus and test conditions

The measurements were made at $M = 9$ in the Imperial College no. 2 Gun Tunnel using nitrogen as the test gas. The design, operation and performance of the tunnel have been fully described in the paper by Needham, Elfstrom & Stollery (1970). The sharp-edged flat plate was inclined at various angles of incidence α between 0 and 26·5° in order to generate local Mach numbers ranging from 9 to 3. The complete range of equivalent free-stream conditions is given in table 1, where the subscripts ∞, 0, e and w refer to free-stream, reservoir, boundary-layer-edge and wall values respectively. Re_x is the Reynolds number based on the edge conditions and length along the plate. Two plates were used and for forced transition a row of delta-wing vortex generators, 1 mm high and with 3 mm pitch, were placed 10 mm from the leading edge.

The two flat-plate models used in the tests were 30 and 76 cm long with spans of 13 and 18 cm respectively. The smaller model was used for the higher (15° and 26·5°) incidence tests to enable the most efficient use to be made of the uniform

	α (deg)	M_e	$Re \times 10^{-5}$ (cm^{-1})	Max Re_x $\times 10^{-6}$	Trip fitted
$M_\infty = 9{\cdot}22$, $Re_\infty = 4{\cdot}7 \times 10^5\ cm^{-1}$, $T_0 = 1070°$ K, $T_w = 295°$ K, $T_\infty = 64{\cdot}5°$ K	0	9·22	4·7	34·2	No
	0	9·22	4·7	34·2	Yes
	7	7·15	6·85	35·2	No
	7	7·15	6·85	36·0	Yes
	15	5·1	6·22	11·7	No
	26·5	3·08	3·8	10·7	No
$M_\infty = 9{\cdot}08$, $Re_\infty = 2{\cdot}28 \times 10^5\ cm^{-1}$, $T_0 = 1070°$ K, $T_w = 295°$ K, $T_\infty = 65{\cdot}3°$ K	0	9·08	2·28	16·6	No
$M_\infty = 8{\cdot}96$, $Re_\infty = 1{\cdot}2 \times 10^5\ cm^{-1}$, $T_0 = 1070°$ K, $T_w = 295°$ K, $T_\infty = 65{\cdot}5°$ K	0	8·96	1·2	8·7	No
	0	8·96	1·2	8·7	Yes
	7	7·00	1·83	9·2	No
	15	5·00	1·54	2·9	No

TABLE 1. Test conditions for heat-transfer tests

tunnel test core. Both models were instrumented along the centre-line with closely spaced thin-film platinum-on-glass surface temperature gauges.

The surface temperature histories during the 10 ms run were converted to measure the heat-transfer rate by electrical analog circuits and the outputs recorded on oscilloscopes. The plates were of low aspect ratio so the flow along the centre-line was expected to depart from being two-dimensional somewhere upstream of the trailing edge. To measure the edge effects some tests were made at the largest plate incidence (lowest local Mach number, $M_e = 3$) with and without side plates. The leading edges of the side plates were swept back 70°. The leading-edge shock angle was at 8° to the plate and hence lay well within the side plates. Pressure and heat-transfer-rate records were taken along the centre-line and at various spanwise stations.

4. Theory

Six prediction methods were used ranging in complexity from a simplified version of Eckert's reference enthalpy method to a two-equation turbulence model used with the Patankar–Spalding finite-difference scheme. The six methods fall into two broad groups. The four in group 1, Eckert (1955), Spalding & Chi (1964), Van Driest (1956) and Fernholz (1971), all predict C_f as a function of Re_θ. The two in group 2, Green (1972) and Gibson & Spalding (1971), predict St as a function of Re_T directly.

In order to compare the predictions of the group 1 methods with our experimental data a value of the Reynolds analogy factor F is needed. Unfortunately there is a large scatter in the measured values of F, especially under high Mach number, cold-wall conditions. Here we have used a value $F = 1$ for the group 1 predictions, so that

$$St = \tfrac{1}{2}C_f \tag{4}$$

and

$$Re_\Gamma = Re_\theta(H_r - H_w)/(H_e - H_w). \tag{5}$$

The first three methods listed are all very well known but in simplifying Eckert's method we have used the Blasius incompressible value for C_f and assumed that $\mu \propto T^{0.76}$. The result is that

$$C_f = 0.026 Re_\theta^{-\frac{1}{4}}(T^*/T_e)^{-0.81}, \tag{6}$$

where T^* is the reference temperature. In the more recent correlation by Fernholz, C_f is expressed as a function of a momentum-thickness Reynolds number based on the viscosity at the wall. Different formulae are given depending on whether the Mach number M_e is above or below 4·5 and whether $Re_\theta(\mu_e/\mu_w)$ is above or below 2000. For further details the reader is referred to the original paper.

The second group of methods predicts St *vs.* Re_Γ directly, so the user is not involved in choosing a suitable value for F. In a recent report, Green (1972) has extended the entrainment method of Head to compressible flow and in an appendix suggests a further extension to non-adiabatic walls. In this extension both the momentum-integral and energy-integral equations together with the entrainment equation are solved simultaneously using the Runge–Kutta method. The skin-friction law employed by Green is 'tied' to the flat-plate Spalding–Chi correlation. It should be emphasized that the heat-transfer rate is not obtained from C_f by use of an assumed F, but is calculated by solution of the energy-integral equation.

The finite-difference scheme used was that of Gibson & Spalding (1971), who modified the basic Patankar–Spalding (1970) program for use with the k–W model of turbulence for boundary-layer flows. In the k–W model the Reynolds stress is expressed in terms of an effective viscosity which is given in terms of the turbulent kinetic energy k and a quantity W related to the time-mean square of the vorticity fluctuations. Separate differential equations are used for k and W, so that, with the momentum and energy equations, there are four partial differential equations to solve. The skin-friction coefficients and Stanton numbers are found from modified log laws developed from a Couette-type analysis in the wall region. Full details are given in the quoted references. Gibson's program was modified slightly to cope with the input data.

Both the integral technique and finite-difference scheme require initial conditions and starting profiles. A simple power-law profile and Crocco temperature distribution were assumed for both methods with the k and W starting profiles suggested by Gibson. Both methods were started at an artificially low Re_θ so that the choice of power-law index (varied from $\frac{1}{7}$ to $\frac{1}{12}$) was no longer significant by the time the first experimental point was reached.

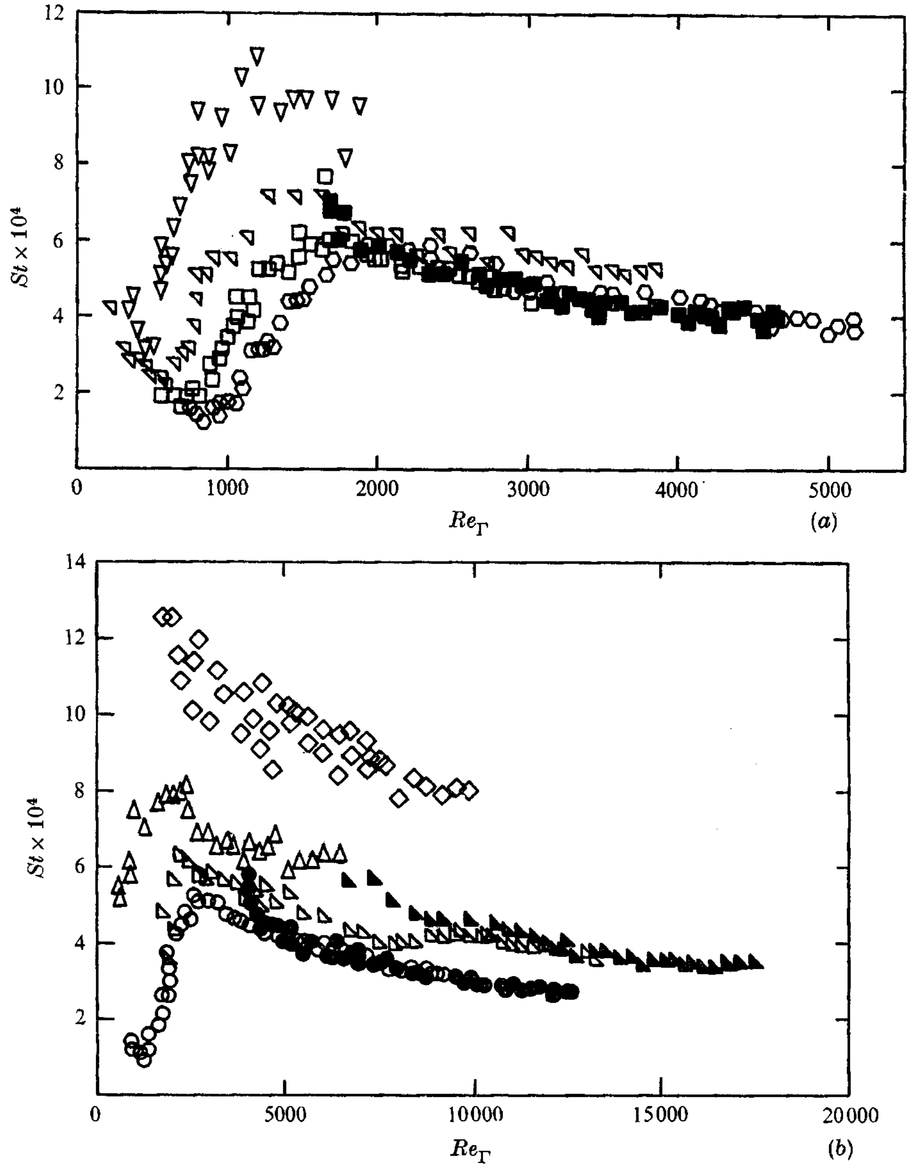

FIGURE 1. Stanton number as a function of the energy-thickness Reynolds number, for all the sets of data indicated in table 1. (*a*) ▽, $M = 5{\cdot}0$; ◁, $M = 7{\cdot}0$; □, $M = 8{\cdot}96$; ■, $M = 8{\cdot}96$, forced transition; ⬡, $M = 9{\cdot}08$. (*b*) ◇, $M = 3{\cdot}08$; △, $M = 5{\cdot}1$; ◺, $M = 7{\cdot}15$; ◣, $M = 7{\cdot}15$, forced transition; ○, $M = 9{\cdot}22$; ●, $M = 9{\cdot}22$, forced transition.

5. Results and discussion

The raw data ($\dot{q}$ *vs.* x) are tabulated in the paper by Coleman (1972). Figure 1 shows this data reduced to the form St *vs.* Re_Γ. The strong dependence on Mach number is clear. Figure 2 compares our data with some measurements made in the British Aircraft Corporation supersonic tunnel at Warton by Eaton *et al.*

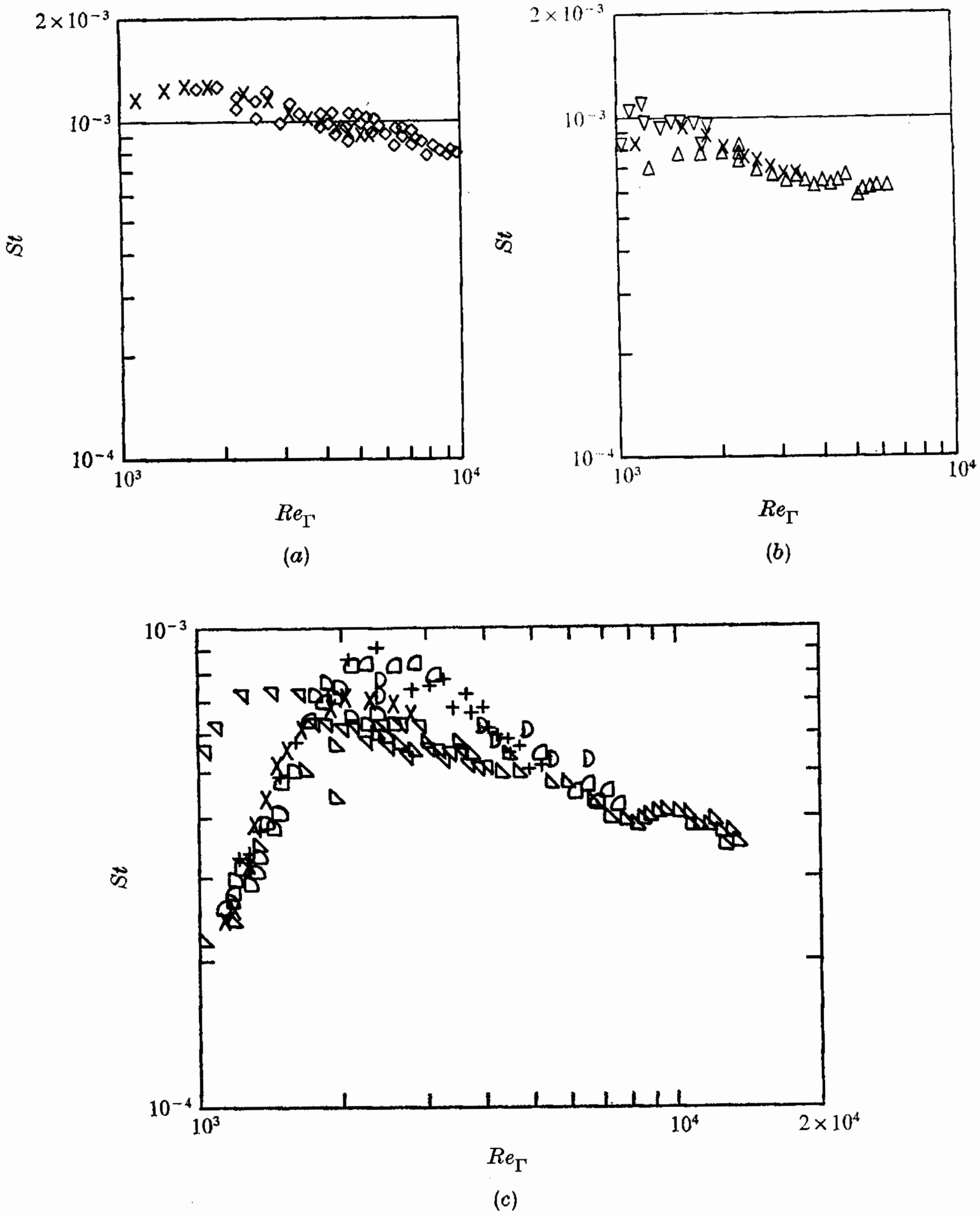

FIGURE 2. Comparison of present results with data from other sources. (*a*) ◇, $M = 3{\cdot}08$, $T_w/T_0 = 0{\cdot}28$, present study; ×, $M = 3{\cdot}00$, $T_w/T_0 = 0{\cdot}75$, Eaton *et al.* (1969). (*b*) ▽, $M = 5{\cdot}0$, $T_w/T_0 = 0{\cdot}28$, present study; △, $M = 5{\cdot}1$, $T_w/T_0 = 0{\cdot}28$, present study; ×, $M = 5{\cdot}02$, $T_w/T_0 = 0{\cdot}73$, Eaton *et al.* (1969). (*c*) ◁, $M = 7{\cdot}0$, $T_w/T_0 = 0{\cdot}28$, present study; ▷, $M = 7{\cdot}1$, $T_w/T_0 = 0{\cdot}28$ present study; ◺, ◿, +, $M = 7{\cdot}0$, $T_w/T_0 = 0{\cdot}42$, Batham (1972); ×, $M = 6{\cdot}8$, $T_w/T_0 = 0{\cdot}30$, Polek (see Hopkins *et al.* 1969); D, $M \simeq 6{\cdot}9$, $T_w/T_\Gamma \simeq 0{\cdot}30$, Hopkins *et al.* (1972) (skin-friction data).

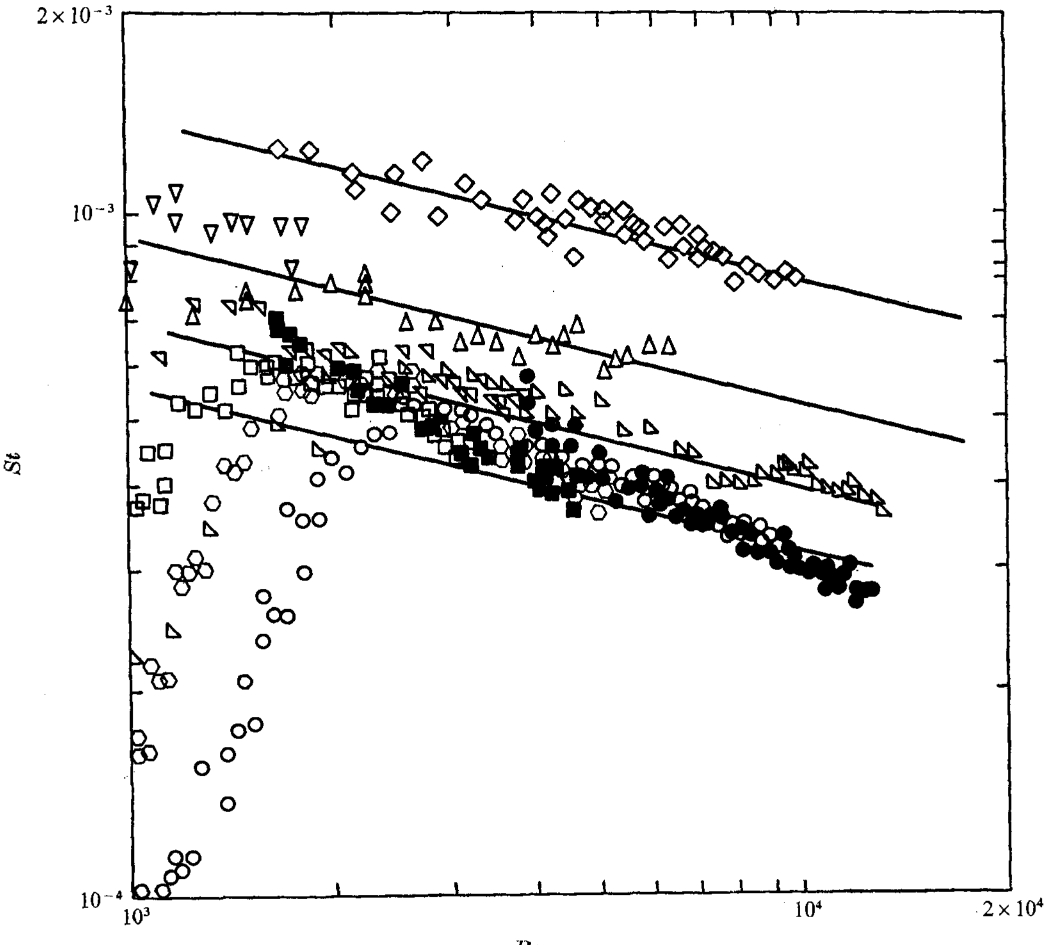

FIGURE 3. Logarithmic plot of all the data from figure 1 compared with lines of gradient $-\frac{1}{4}$. Symbols as for figure 1.

(1969), the data of Polek (NASA Ames) reported by Hopkins *et al.* (1969) and with some of Batham's data (1972) from his tests in the Oxford University gun tunnel. There are significant differences at low values of Re_Γ. Further comparisons are difficult as there are not many cold-wall measurements which extend well into the fully turbulent regime. Hopkins *et al.* (1972) have measured C_f, under cold-wall conditions in the Mach number range 6–8, using a floating-element balance. They suggest a value for F of unity derived with the aid of Polek's heat-transfer data. Accepting this value for F the Hopkins C_f *vs.* Re_θ data at $M = 6{\cdot}9$ have been converted to St *vs.* Re_Γ and plotted on figure 2. Their points lie somewhat above our own data.

A logarithmic plot of our data (figure 3) shows that only the low Mach number ($M_e = 3$) results follow the simple dependence $St \propto Re_\Gamma^{-\frac{1}{4}}$. The deviation from this 'law' seems to increase with Mach number and to be greatest at the lowest values of Re_Γ. This behaviour may well be associated with the slow development

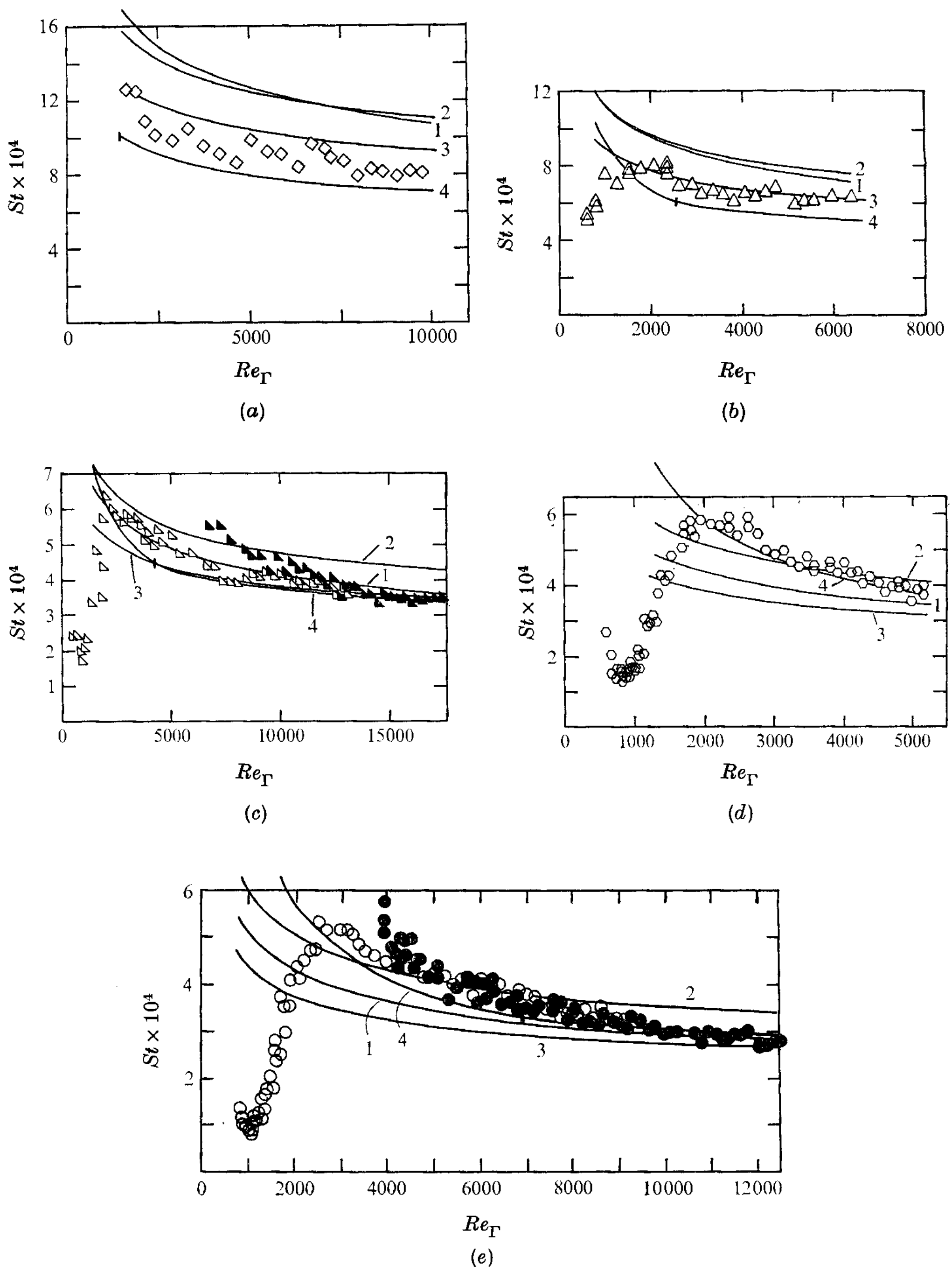

FIGURE 4. Comparison with the group 1 theories. Symbols as for figure 1. Theories: 1, Eckert (1955); 2, Van Driest (1956); 3, Spalding & Chi (1964); 4, Fernholz (1971). The vertical bar indicates for theory 4 where $\rho_e u_e \theta / \mu_w = 2000$. (*a*) $M = 3{\cdot}08$. (*b*) $M = 5{\cdot}1$. (*c*) $M = 7{\cdot}15$. (*d*) $M = 9{\cdot}08$. (*e*) $M = 9{\cdot}22$.

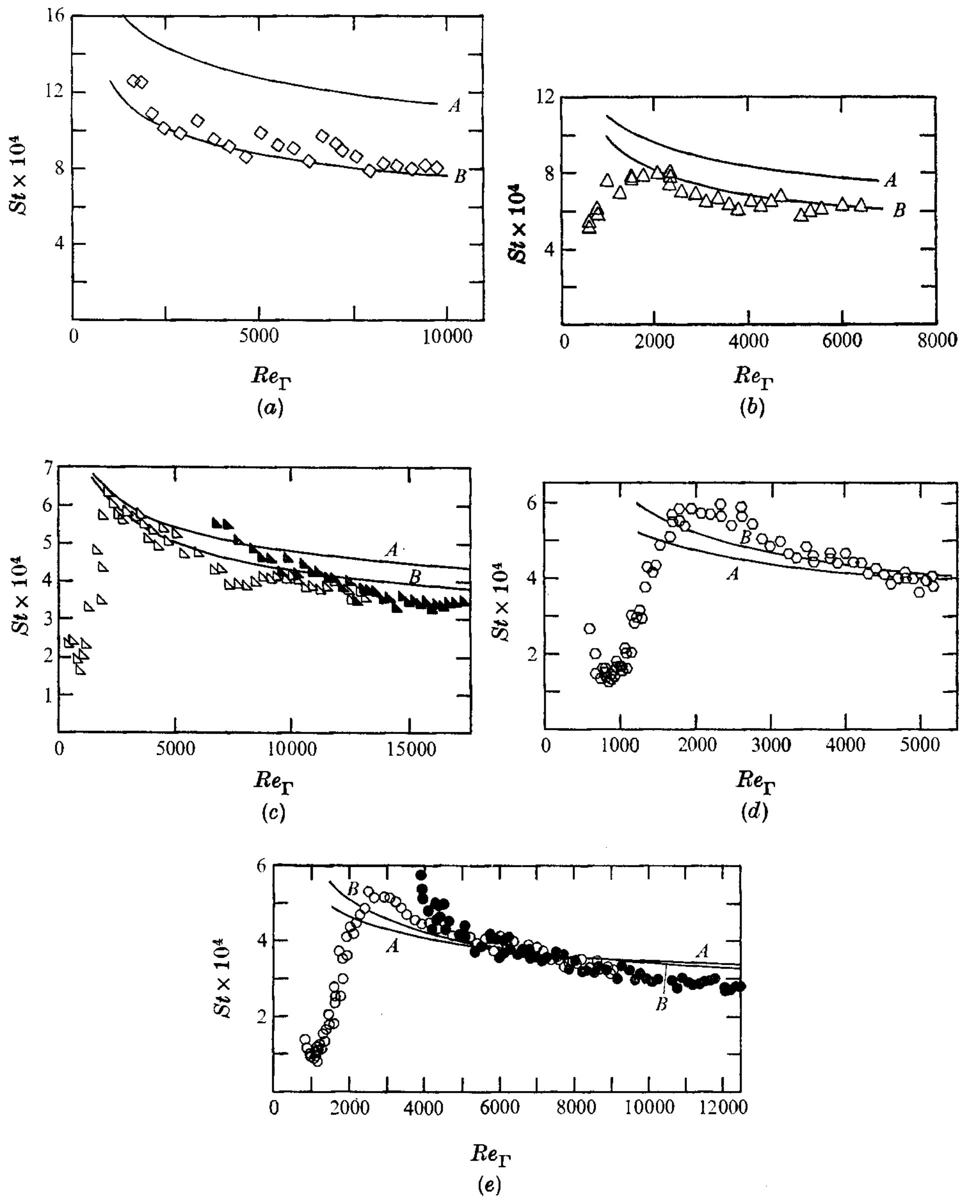

FIGURE 5. Comparison with the group 2 theories. Symbols as for figure 1. Theories: *A*, Green (1972); *B*, Gibson & Spalding (1971). (*a*) $M = 3{\cdot}08$. (*b*) $M = 5{\cdot}1$. (*c*) $M = 7{\cdot}15$. (*d*) $M = 9{\cdot}08$. (*e*) $M = 9{\cdot}22$.

of the wake component in turbulent velocity profiles as reported by Hastings & Sawyer (1970) at $M = 4$. Of the calculation methods used, the only one to recognize this behaviour is that of Fernholz,† which is the main reason why

† It is important to note that this method is primarily empirical and that it predicts a small rise of skin-friction coefficient with a rise in wall temperature ratio. This trend is the opposite of that predicted by nearly every other theoretical method.

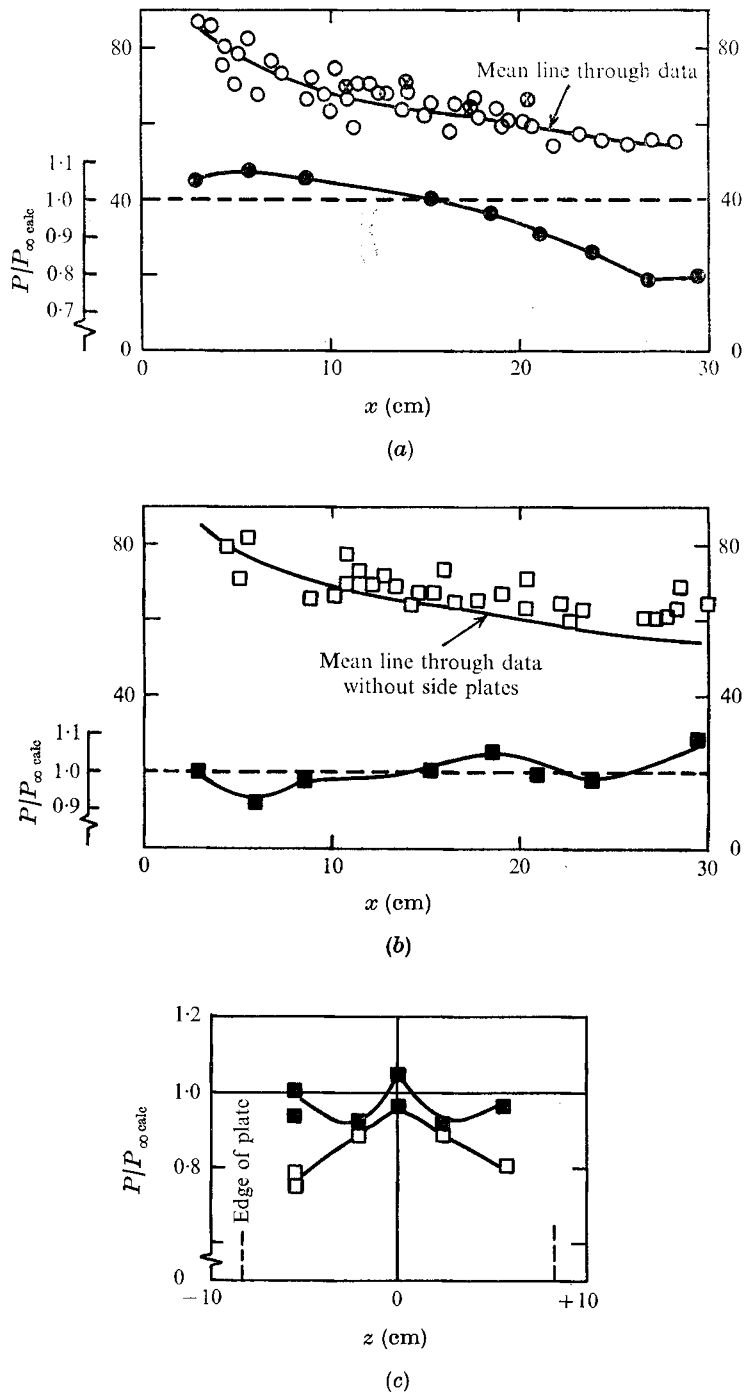

FIGURE 6. The three-dimensional nature of the flow, for the $\alpha = 26{\cdot}5°$, $M = 3{\cdot}08$ case. (a) Without side plates: ○, heat-transfer data; ●, pressure data. (b) With side plates: □, heat-transfer data; ■, pressure data. (c) Spanwise pressure distribution at $x = 18{\cdot}5$ cm: □, without side plates; ■, with side plates.

his suggested formulae give the best overall fit to our experimental data in the range $3 \leqslant M \leqslant 9$. The comparisons are shown in figures 4 and 5. Again it must be emphasized that for the simpler theories (group 1) a value of $F = 1$ has been used even when the authors have suggested a different value. For example, Spalding & Chi suggest a value of 1·16 and this would improve the agreement with our experimental data at $M = 9$ but worsen it at $M = 3$ and 5. However, the main result from the comparison is to show that no single method apart from that of Fernholz gives reasonable agreement throughout the Mach number range and this is the only method to account for the high heat-transfer rate recorded in the early stages of turbulent boundary-layer development, i.e. just after transition.

Hopkins *et al.* (1972) have made similar comparisons between various theoretical predictions on the basis of their skin-friction measurements. Although they concluded that the methods of Van Driest (1956), Coles and Dwyer (finite-difference scheme) all predict their measured values to within 10 %, it should be noted that the variation of C_f with Re_θ for their data is steeper than that given by all the theories except Coles's. Like Fernholz's in the present study, Coles's was the only theory tested which allowed for the low Reynolds number behaviour of the turbulent boundary layer. The sucess of the Van Driest method is of particular interest since, if $F = 1$ and the measurements from both sources were compatible, then the method should also have shown up well in our own comparison. It did not and we conclude either that $F < 1$ or the two sets of experimental data are dissimilar, i.e. each reflects the tunnel environment in which it was made. We can only point out that our data do cover a much wider range of Mach number and Re_Γ, enabling a more critical evaluation of any prediction method to be made.

The results of tests made at $M_e = 3$ ($\alpha = 26°$) to examine the three-dimensional nature of the flow are shown in figure 6. The pressure distribution along the centre-line is reasonably uniform until the Mach lines from the tip reach the centre. Without side plates the pressure then falls towards the trailing edge; however, with side plates fitted the pressure rises, presumably owing to the displacement effect of the side-plate boundary layers. The increase in heat-transfer rate, with side plates, is about 4 %, except very close to the trailing edge of the plate, where it is 12 % (see figures 6*a*, *b*). Thus the edge effects, though not insignificant, do not alter the main conclusions of this note. The spanwise variation of pressure (figure 6*c*) shows the expected decrease in pressure towards the edges and fitting side plates reverses this trend.

Some Mach number profiles derived from Pitot pressure measurements at $M = 9$ show how the profile is fuller at the lower values of Re_Γ (figure 7). The corresponding velocity profiles obtained using the Crocco temperature relation are shown as a Clauser plot with the ordinates suggested by Sivasegaram (1971)† in figure 8. However, there is such a small log-law region that we have little confidence in the value of skin-friction coefficient obtained in this way. Preston tube measurements were also made at $M = 9$ (figure 9). Two tube diameters were used within the range specified by Keener & Hopkins (1969), whose calibration

† Sivasegaram makes a number of suggestions for defining ρ^* and of these we have used $\rho^* = \frac{1}{4}(\rho_w + 2\rho_{\frac{1}{2}} + \rho_{\text{local}})$. Readers are referred to the original paper for details and definitions.

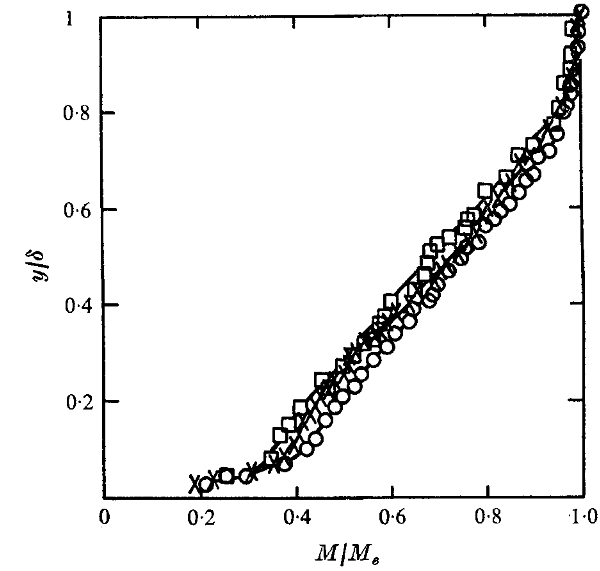

FIGURE 7. Mach number profiles for a turbulent boundary layer over a flat plate at $M = 9$. ○, $Re_\Gamma = 3100$, $M_\infty = 8{\cdot}96$; ×, $Re_\Gamma = 4700$, $M_\infty = 8{\cdot}96$; □, $Re_\Gamma = 12500$, $M_\infty = 9{\cdot}22$.

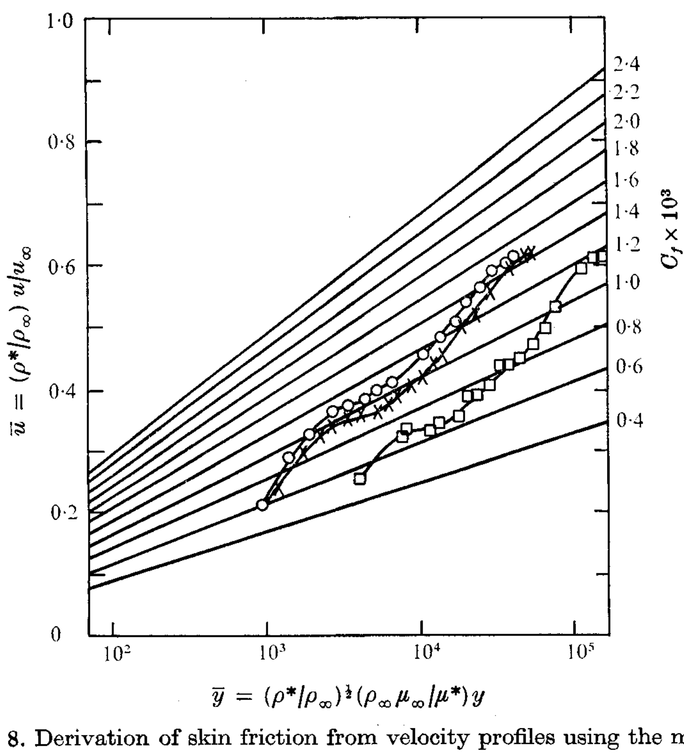

FIGURE 8. Derivation of skin friction from velocity profiles using the method of Sivasegaram (1971). Symbols as for figure 7.

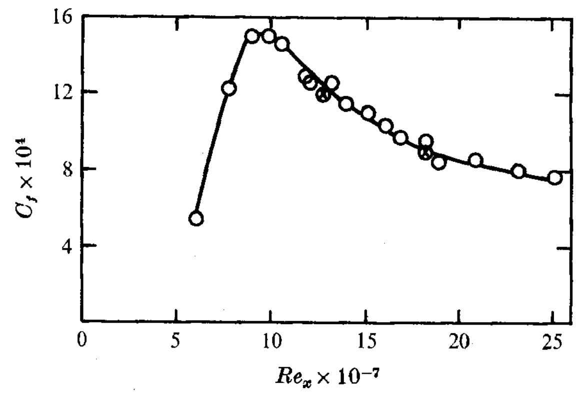

FIGURE 9. Skin-friction measurements using a Preston tube. ○, tube diameter $d = 1{\cdot}52$ mm; ⊗, $d = 2{\cdot}54$ mm.

M_∞	Re_Γ	C_f	St (measured)	F
8·96	3100	$1{\cdot}06 \times 10^{-3}$	$0{\cdot}47 \times 10^{-3}$	0·89
8·96	4700	$0{\cdot}92 \times 10^{-3}$	$0{\cdot}40 \times 10^{-3}$	0·87
9·22	12500	$0{\cdot}66 \times 10^{-3}$	$0{\cdot}30 \times 10^{-3}$	0·91

(*a*)

M_∞	C_f	St	Re_θ	Re_Γ	$F = 2St/C_f$
9·22	$0{\cdot}89 \times 10^{-3}$	—	9400	—	0·85
9·22	—	$0{\cdot}375 \times 10^{-3}$	—	7000	
9·22	$0{\cdot}78 \times 10^{-3}$	—	11700	—	0·86
9·22	—	$0{\cdot}34 \times 10^{-3}$	—	8740	

(*b*)

M_∞	Re_x	C_f	St	F
9·22	22×10^6	$0{\cdot}98 \times 10^{-3}$	$0{\cdot}41 \times 10^{-3}$	0·84

(*c*)

TABLE 2. Comparisons between measurements of the skin-friction coefficient and Stanton number. (*a*) C_f from Clauser plots of the velocity profiles using the method proposed by Sivasegaram. (*b*) C_f from Preston tube measurements using the calibration of Keener & Hopkins by comparison of the St *vs.* Re_Γ and C_f *vs.* Re_θ curves (θ and Γ were found by integrating the measured distributions of C_f and St respectively). F is calculated as $2St/C_f$ using local values of C_f and St at points where Re_θ and Re_Γ also satisfy the relation $F\,Re_\theta(H_r - H_w) = Re_\Gamma(H_e - H_w)$, assuming that the local value of F does not significantly vary from the average value over the plate. (*c*) C_f from Preston tube measurements using the calibration of Keener & Hopkins by measurements of C_f and St at the same value of x.

was employed to calculate the values of C_f. Table 2 compares the experimental values of C_f and St. They suggest a value for F of 0·85, which is lower than commonly assumed values. Although Hopkins (private communication) claims $\pm 5\,\%$ accuracy for his Preston tube calibration up to $M = 7{\cdot}4$ the seemingly low value of F at $M = 9$ suggests that our skin-friction values are too high. It may be that a skin-friction balance is the only way of obtaining accurate data at the present time.

6. Conclusions

Experiments have been made which increase the range of hypersonic turbulent boundary-layer heat-transfer data available for testing flat-plate prediction techniques. A number of prediction methods were compared with these data. For the methods of group 1 an F of 1 was used, although the original authors advocate higher values. The main discrepancy between theory and experiment is the steeper variation of St with Re_T at the higher Mach numbers, which seems to be a low Reynolds number effect. Only Fernholz allows for this effect and his method is overall the most successful in predicting both the level and trend of the data in the Mach number range $3 \leqslant M \leqslant 9$. At the lower Mach numbers of 3 and 5 all the methods predict the correct trend of St with Re_T but differ considerably in magnitude, the Gibson–Spalding method being the most accurate.

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